The Relationship between Actuarial Inputs and the Financial Condition of Public Pensions

Abstract

There are significant concerns about the financial condition of public pensions in U.S. state and local governments. A growing number of observers contend that those conditions are actually much worse than government financial reports currently indicate. At the root of those claims is a view that governments use overly optimistic and risk-accepting actuarial inputs that allow them to project lower pension liabilities than is financially prudent. This paper uses simulated pension system data to examine the effect of actuarial inputs on the funding ratios and contribution requirements of public pensions. Our findings demonstrate the significant effect of discount rates, salary growth rate assumptions, cost methods, amortization periods, and mortality tables and important interactive effects among the inputs. We also find that the effect of actuarial inputs is greatly influenced by the demographic profile of system participants and by asset levels.

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Introduction

Public pensions are at the center of many contentious policy debates in U.S. state and local governments (Chapman 2008). The salience of those debates, the persuasiveness of relevant policy options, and the political urgency for policy change is greatly influenced by the size of unfunded pension liabilities and the budgetary burdens those liabilities create. Some argue that unfunded liabilities are becoming so large that they are harming state and local governments by threatening their long-term financial solvency, crowding-out resources for other public services, necessitating tax increases, and damaging governments’ ability to borrow capital resources. That position lends support to those who want to reform public pensions by reducing pension benefits and potentially closing traditional pension plans in favor of defined contribution plans.

Given the importance of unfunded liabilities in public policy debates, it should not be surprising that there is substantial disagreement on the scope of the problem. The Pew Center on the States (2012), for example, used government reports to estimate that the total size of unfunded state pension liabilities in 2010 was $757 billion. A few years later, Munnell et al. (2013) also used government reported figures to estimate the total unfunded liability to be about $1 trillion. Government reports are widely accepted. The American Federation of State, County and Municipal Employees (AFSCME 2014) supports claims that total unfunded liabilities are between $500 billion and $1 trillion.

Others disagree with the way governments calculate their liabilities and claim that unfunded liabilities are significantly higher. Novy-Marx and Rauh (2011) used more conservative actuarial assumptions (i.e., lower discount rates) to estimate that unfunded liabilities are closer to $3 trillion. Biggs (2012) used a market-value approach to put the total unfunded liability at approximately $4.6 trillion. The significant disparity between government figures and those calculated by Novy-Marx and Rauh (2011) and Biggs (2012) shows the effect that calculation methods have on measures of the financial condition of public pensions.

Different liability calculations are possible because the actual size of pension liabilities is fundamentally unknowable. As such, liability calculations are dependent on the assumptions that are selected to forecast future benefits and then to discount those future payments to their present value. That process is performed by actuarial professionals under the direction of pension system trustees and within legislative parameters. Actuaries also use cost methods and amortization periods to determine how much sponsors need to contribute each year in order to adequately fund their liabilities. The selection of assumptions and procedures greatly affects the size of reported liabilities and contribution requirements and, by extension, the substance of pension policy debates. In this paper, we use the term "actuarial inputs" to refer to the assumptions and procedures used by actuaries to value pension liabilities and contribution requirements.

When plan sponsors select actuarial inputs that reduce their pension liabilities and contribution requirements, they ease the short-term fiscal burden of public pensions and make it easier for advocates of more generous pension benefits to persuade policy makers to increase and expand benefits provisions. There is, therefore, a clear short-term incentive for governments and public employee advocates to support more optimistic actuarial inputs. These optimistic calculations, however, do not actually improve pension funds’ ability to pay future benefits; so that strategy only works in the short-term and creates potentially unsustainable costs as employees' near retirement.

In contrast, selecting more conservative actuarial inputs increases the assumed liabilities and contribution requirements. This incentivizes policy actions that reduce benefits, limit the number of annuitants, and potentially even close defined-benefit systems in favor of defined-contribution plans.

While the deterministic relationship between actuarial inputs and pension liabilities and contribution requirements is well established and the direction of those relationships is well known, there are several gaps in our understanding of the relationship. *First*, the policy and administrative community knows little about the magnitude of the effect that a marginal change to actuarial inputs creates on unfunded liabilities and contribution requirements. Much of the previous analysis greatly simplifies the calculations by not accounting for important actuarial inputs such as mortality tables and actuarial cost methods (Peterson 2004). Others have focused more on the effect of changes in asset values on the financial condition of pension plans (Dufresne 1988). The lack of information on the marginal effects of actuarial inputs means non-financial professionals have limited understanding of the pressure governments may be under to maintain assumptions after they are discovered to be overly optimistic or to select more optimistic assumptions in order to ease budgetary constraints. *Second*, most of the current attention on actuarial inputs is focused on the discount rate (e.g., Anson 2011, Brown et al. 2011, Novy-Marx and Rauh 2011). We know little about the importance of other actuarial inputs, such as salary growth rates, amortization periods, cost methods, and mortality rates. *Third*, there is little information on how actuarial input changes are affected by plan characteristics, such as the demographic profile of plan participants (i.e., plans where the majority of the participants are young versus those where the major of participants are nearing retirement) and the funding condition of pension plans (i.e., well-funded plans versus poorly funded plans).

This paper seeks to improve our understanding of the public-pension policy debate by providing information to narrow those three gaps in the literature. This paper also seeks to expand our understanding of incentives and conditions that may affect the managerial decision-making framework in pension systems (Peng 2004). We do this by simulating pension plans with different age structures and asset levels, and then calculating the change in liabilities and contribution requirements for different values of five actuarial inputs (discount rates, salary growth rates, mortality tables, cost methods, and amortization periods).

The paper proceeds as follows. In the next section, we discuss the actuarial process. Then, we describe the relationship between actuarial inputs and pension management. We then discuss each of the actuarial inputs examined in this study. Then, we present the simulation and its results. We conclude the paper with a discussion of key findings, study limitations, and suggestions for further research.

The Actuarial Process

Pension actuarial science is the process of evaluating assets and liabilities in a pension plan (Ezra 1980). Liabilities are retirement payments to qualified pension plan participants and their beneficiaries. From a plan sponsor’s perspective, the actuarial process helps (1) calculate the present value of their future obligations and (2) develop a savings strategy that allows them to smooth their budgetary contributions over the years when employees earn their pension benefits.

This process is riddled with uncertainty (Ezra 1980). Sponsors respond to that uncertainty by making important assumptions about the size of their future obligations, the likelihood that benefits will need to be paid, and how much of the obligations should already be funded in order to have all the necessary assets set aside by the time employees retire. As workers near retirement, the future becomes more certain and assumptions are less critical. But, once benefits become certain (or even as they near certainty), it is generally too late to make up a significant deficit in pension funds. Plan sponsors who set assumptions that turn out to be too optimistic will not have saved enough to pay their retirees' benefits from pension fund savings; and, in order to make up the short fall, sponsors will need to increase their annual contributions and/or consume pension fund assets that were intended for future retirement payments.

In an effort to avoid funding shortfalls, actuaries use two sets of assumptions to calculate pension liabilities and contributions requirements (American Academy of Actuaries 2004). *Demographic assumptions* are used to predict employee turnover and the number of workers who will qualify for pension benefits. Demographic assumptions include mortality rates, turnover rates, disability rates, and termination rates. *Economic assumptions* are used to forecast growth in the future value of pension benefits and to calculate the present value of those future payments. Economic assumptions include inflation rates, discount rates, and salary growth rates.

Actuaries use those assumptions to calculate pension liabilities and contribution requirements. *Pension liabilities* represent the present value of future benefit payments that have already been earned by employees. The pension liability for a plan with multiple participants is calculated as the sum of the individual liabilities for each qualified plan participant. *Contribution requirements* represent the actuarially determined amount of money that needs to be deposited each year into a pension fund and held in trust for future benefit payments.

The two measures are highly related. One portion of the contribution requirement is the amount of money needed to cover the benefits that are earned in the current year (the normal cost (NC)). Pension liabilities, or the actuarially accrued liability (AAL), are the accumulation of normal costs for all the completed years of employees’ service.

When pension funds do not hold sufficient assets to cover their AAL, the plan recognizes an unfunded actuarial accrued liability (UAAL). Plans with UAALs are not expected to make up the shortfalls in a single year. Rather, they are allowed to finance the unfunded liability over a period of time (an amortization period). The amortized UAAL is the second portion of sponsors’ contribution requirement.

The combination of the normal cost and the amortized UAAL is usually referred to as the annual required contribution or ARC. It is often expressed in terms of its percent of covered employees’ payroll. New accounting standards refer to this calculation as the actuarially determined contribution (GASB 2012), though we retain the common ARC terminology in this paper.

The actuarial process assumes that conditions will change and assumptions will need to be revised. Actuaries, therefore, conduct periodic reviews of plan conditions, usually no less frequently than every five years, and test whether the actuarial assumptions are consistent with actual experience (GFOA 2009; Peng 2008). Actuaries regularly suggest changes to actuarial inputs based on (1) whether actual conditions deviate from actuarial assumptions, (2) changes in professional standards (e.g., revised mortality tables), and (3) changes in governmental accounting standards. Actuaries may suggest revisions, but boards of trustees and sponsoring governments’ governing bodies are responsible to review those suggestions and determine whether to revise actuarial inputs.

Actuarial Inputs and Pension Policy

Actuarial practices in the public sector have been highly criticized over the past century. In the early 1900s, most pension plans did not use an actuarial process to fund their pension obligations. Rather, most plans were financed using the pay-as-you-go method, where retiree benefit payments are paid from budgetary resources rather than using an actuarial process to “prepay” or “prefund” pension liabilities (Clark, Craig and Sabelhaus 2011). Through the early- to mid-part of the twentieth century, the number and size of public pension systems continued to grow while accrual practices continued to lag behind professional standards. In 1978, the congressional *Pension Task Force Report on Public Employee Retirement Systems* found a “high degree of pension cost blindness [...] due to the lack of actuarial valuations, the use of unrealistic actuarial assumptions, and the general absence of actuarial standards (1978, p. 4).”

The adoption of governmental accounting standards for pension reporting in the 1980s and 1990s (GASB 4 in 1986 and GASB 27 in 1994) improved the transparency of pension obligations in state and local governments (Dulebohn 1995). Those standards, however, still permitted significant discretion in actuarial inputs to the point that many questioned the validity of the pension figures in governmental financial reports. In recent years, these criticisms have led to congressional bills that have attempted to require state and local pensions to report their liabilities using a standard set of actuarial assumptions, especially mandating that discount rates reflected a so-called riskless rate of return (see H.R. 567 in 2011 and H.R. 1628 in 2013); similar bills were also introduced at the state level (see Florida SB 534 in 2013). The criticisms of actuarial practices, also led Moody’s Investor Service to implement its own actuarial process for valuing pension obligations when setting and revising municipal bond credit ratings (Moody's 2013).

The newest pension accounting standard, GASB 68, was recently issued and is effective for employer fiscal years beginning after June 15, 2014. Among its provisions, the standard specifies the need to use the Entry Age Normal (EAN) actuarial cost method (EAN front loads the recognition of retirement obligations compared to the next most common method, Projected Unit Credit (PUC)) and places constraints on the use of higher discount rates by pension plans that are deeply underfunded. However, GASB 68 still permits a significant range of actuarial inputs. GASB 68 also moves away from setting standards on contribution requirements.

Academic studies suggest that decision makers have used the discretion allowed by accounting standards to adopt actuarial inputs that postpone pension costs in order to spend more on current government services (Johnson 1997; Marks, Raman and Wilson 1988). Those incentives are thought to be particularly strong in governments under budgetary stress (Chaney, Copley, and Stone 2002; Giertz and Papke 2007; Mitchell and Hsin 1997; Schneider and Damanpour 2002). Other studies find that the selection of actuarial inputs is affected by political pressure (Romano 1993). For example, Levine, Rubin and Wolohojian (1981) found that when the City of Oakland was under fiscal stress in the 1970s, their pension system increased its amortization period from 14 years to 40 years. Also, Eaton and Nofsinger (2004) found that governments under political pressure and fiscal constraints, decreased their salary growth rate assumptions, increased their amortization periods, and increased their discount rates. And, Chaney et al. (2002) found that states with balanced budget requirements increased their discount rates when facing fiscal stress.

Anecdotal evidence supports claims that actuarial assumptions are sometimes changed in order to improve short-term fiscal and political conditions. In response to a fiscal crisis in the early 1990s, the State of New York changed their method of allocating the costs of pension obligations (Bentley 2009). New York's public employees challenged that decision, and the court ruled against the state, deciding that the change was not intended “to protect the retirement funds earned by members and beneficiaries of the system but to deal with the budget crises being experienced by governmental entities in New York State.” (McDermott v. Regan 1993). In 2009, the City of Philadelphia announced a plan to reduce its ARC by $170 million by increasing its amortization period from 20 to 40 years (Lucey 2009). And, a member of the Montana Board of Investments cited significant gubernatorial pressure as a reason why the Board reversed its decision to reduce the Montana Public Employees Retirement Systems' discount rate (Johnson 2013).

Actual Variation in Pension Assumptions

While there are strong modal norms in the selection of actuarial inputs by public pension systems, there is still a fair amount of variation around those norms. In this paper we focus on five actuarial inputs: discount rates, salary growth rates, amortization periods, cost methods, and mortality tables. In this section, we describe each of those actuarial inputs and summarize their variation in state and large locally-administered pension systems. We identify the distribution of actuarial inputs in U.S. pension systems by using the Center for Retirement Research at Boston College's Public Plans Database (PPD) (Public Plans Database 2001-2011). The PPD provides information on 126 state and local pension plans from 2001 to 2011 and includes more than 85 percent of state and local pension assets and membership.

Tables 1 and 2 demonstrate the variation in actuarial inputs in public pensions. Three qualifications are important to note. First, the range of values in table 1 excludes the outliers at the minimum and maximum positions in order to more accurately present the range of typical values in practice. Second, mortality tables are not reported in the PPD and are therefore not shown in tables 1 or 2. Third, the discount rate and salary growth rate assumptions reported in table 1 include the inflation rate assumptions; inflation rate assumptions are not discussed explicitly in the rest of this paper, but they are implied by the discount rate and salary growth rate assumptions.

<Table 1>

<Table 2>

***Discount rates*** are used to calculate the present value of future benefits. Larger discount rates lead to lower liabilities, higher funding ratios (i.e., the ratio of asset holdings to liabilities), and lower contribution requirements. Accounting standards hold that discount rates should reflect the expected rate of return on the investment of pension assets (GASB 1994a, 1994b). GASB 27 required that discount rates are based on the “estimated long-term investment yield” but did not specifically list acceptable rates.

As reported in table 1, the median discount rate is 8 percent and the range of typical discount rates is between 6 and 8.75 percent. This reflects an explicit assumption by most governments that long-term returns from pension fund investments will average about 8 percent.

The "expected rate of return" standard is widely criticized by financial economists who contend that discounting liabilities should not be based on asset growth expectations but, rather, that discount rates should reflect the near certainty of guaranteeing pension benefits. That perspective leads to discount rates that are representative of risk-less rates of return at maturity horizons that are similar to the number of years that pension systems are obligated to pay their benefits (Novy-Marx and Rauh 2009, 2011). In recent market conditions, this may suggest discount rates that are about five percent (Munnell, et al. 2010).

GASB 68 permits the use of discount rates that reflect the expected rate of return on long-term investments, so long as the net positions of pension funds are sufficient to cover projected benefit payments. It is not known, however, how plan sponsors and auditors will judge whether their assets are sufficient to cover projected benefit payments. Some expect that most plan sponsors will decide that they have the ability to cover their benefit payments and will keep their current discount rates. If sponsors decide they do not have sufficient assets, they need to calculate a blended discount rate that is based on a weighted average of long-term and short-term yields. Blended rates will be lower than the usual 8 percent, but how much lower is unknown.

***Salary growth rates*** are used to predict the geometric growth in employee salaries. Most pension plans calculate pension benefits as a function of (i) years of service, (ii) a measure of employees’ average salaries (usually the average of the highest three to five years of salary income), and (iii) a benefit multiplier. For example, a public employee with 30 years of service and whose final average salary is $85,000 with a 2 percent benefit multiplier will qualify for a $51,000 annual annuity in retirement (30 × $85,000 × .02).

Higher salary growth rates increase the predicted final average salaries. This is expected to lead to higher pension liabilities, lower funding ratios, and higher contribution requirements. Table 1 shows that the typical range in salary growth rate assumptions is 3 to 8 percent and the median assumption is 6 percent. Neither GASB 27 nor 68 provide numerical standards for the selection of the salary growth rate assumption, though GASB 68 does require that future benefit payments consider expected future salary changes.

***Mortality tables*** (not shown in tables 1 or 2 because they are not reported in the PPD) are used to project the likelihood that employees will remain in the plan long enough to receive their benefits and to evaluate the expected life expectancy after retirement. With an aging population and the retirement of baby boomers, there are concerns that some public pension systems calculate their liabilities based on outdated mortality tables. Because life expectancies continue to increase, the most recent mortality tables tend to increase the expectation that employees will survive long enough to receive their retirement benefits and then receive more benefit payments after retirement.

Accounting standards require sponsors to disclosure information on the mortality table they use to value their liabilities, but those standards do not specify what mortality table should be used. Based on our review of state pension systems’ annual reports, most plans adopt mortality tables produced by the Society of Actuaries (SOA), often with adjustments based on plans' experiences. Our simulation looks at three SOA mortality tables: RP-2000, RP-2000 with 10 years projection using scale AA, and RP-2014. The first two SOA mortality tables are widely used by public pension systems. For example, the Illinois Teachers Retirement System uses RP-2000 with a nine-year projection using scale AA (Illinois TRS 2013). Georgia state pension systems use RP-2000 with adjustments (Georgia ERS 2013). The third SOA mortality table, RP-2014, was published in February 2014 as an exposure draft, and will likely be adopted, or considered for adoption, by public pension systems in the coming years.

***Actuarial cost methods*** are used to determine the normal cost for each year of employee service. Cost methods can allocate the *benefits* over employees’ service periods, such as with Projected Unit Credit (PUC), or allocate the *costs* over service periods, such as with Entry Age Normal (EAN) and Aggregate Cost (AGG) (Peng 2008). This distinction is technical, but its effect is that cost-prorated methods (such as EAN) tend to have larger normal costs in early years of employee service, while benefit-prorated methods (such as PUC) tend to produce progressively increasing normal costs.

As shown in table 2, 72 percent of plans use EAN, 13 percent use PUC, eight percent use Aggregate Cost (AGG), and seven percent use other methods (such as Frozen Initial Liability). GASB 27 permits six different actuarial cost methods (with additional variations).

Under GASB 68, only the EAN cost method is allowed for valuing the pension liability. The updated standard, however, permits significant flexibility in reporting contribution requirements. Governments are allowed to report statutory or contractual reporting levels that are not necessarily based on actuarial determined values or may be based on actuarial determined values that are calculated with cost methods other than EAN.

***Amortization periods*** are the time periods over which pension systems finance (i.e., seek to pay off) their unfunded accrued liabilities (UAALs). When amortization periods increase, the annual share of UAALs is reduced. Thus, the choice of amortization period affects systems’ contribution requirement, especially for systems with large UAALs. The amortization period, however, does not affect the liability calculation, and so does not affect the funding ratio in the near term.

Table 1 shows that amortization periods range from 1 to 100 years with a median of 29 years. GASB 27 allowed the maximum amortization period of 30 years (to ease the standards implementation, GASB 27 permitted a maximum period of 40 years in the first 10 years after the standard was issued). GASB 68 requires an amortization period that is based on the remaining service life of all employees, which in many cases is much shorter than the 30 years.

Research Design

**Baseline Simulation Model (The “Median Plan”)**

In order to identify the marginal effects of changes to actuarial inputs on funding ratios and contribution requirements, we simulated a simplified public pension system. We use a simulation over a multivariate-regression approach because of several problems related to the availability of data on actuarial inputs and changes to those inputs. First, the relationship between changes to actuarial inputs and pension performance measurements is likely to be highly endogenous. This means that governments may change their actuarial inputs in response to changes in funding ratios or contribution requirements that were caused by other factors. This can lead to regression findings, for example, that discount rates are negatively related to funding ratios when the discount rate was increased because the funding ratio dropped as the result of asset value declines or benefit increases. Second, there is significant difficulty in identifying the marginal effects of individual actuarial inputs when they change simultaneously with other factors and there are not enough observations to gain enough statistical power to isolate the marginal effects of individual actuarial inputs. Third, governments rarely change some actuarial inputs, such as mortality tables, so even a ten-year time series may not be enough time to obtain a significant number of observations of input changes. Fourth, the marginal effects of actuarial inputs greatly depend on the inactive relationships of other input values, and there are only a limited number of pension systems to observe all the possible combinations of those values.

Simulations have been used in prior research on pension management. For example, Winklevoss & Associates built the Pension Liability and Asset Simulation Model (PLASM) to inform pension plan managers about possible effects of decisions in pension benefit, funding and investment policies (Winklevoss 1982). Kingsland and Gruber (1982) used a simulation model to examine the impacts of investment policies on the financial condition of pension systems. Fehr, Kallweit and Kindermann (2012) simulated the macroeconomic, welfare and efficiency consequences of increasing normal retirement age in the German national pension system. To our knowledge, however, no similar work has been done to systematically examine the effects of changes in various actuarial inputs or to inform current policy debates on public pension systems. Our research fills this gap.

Since actuarial inputs have deterministic effects on pension liabilities and contribution requirements, it is possible to simulate the relationships under hypothetical conditions with a reasonable level of external validity. Also, by isolating the deterministic effects in a controlled environment, there is no need for statistical analysis to determine whether or not differences are the result of random error.

The conditions in our baseline simulation match the median conditions of pension systems in the Public Plans Database (PPD). We call this baseline simulation the “median plan.” We then identified the potential ranges of actual inputs by looking at the observed range of actuarial inputs in the PPD in order to identify the size of potential changes to the liabilities and contribution requirements for the median plan. As mentioned above, we use the 1st and 99th percentile values, rather than the minimum and maximum values, in order to avoid significant outliers.

The basic parameters for the median plan are:

* 108 active employees and 54 retirees (retirees are approximately 1/3 of total participants). The number of employees and retirees is the result of including sufficient number of participants in each age category to vary demographic profiles. Since the focus of the analysis is on funding ratios and contribution requirements as a share of payroll, rather than raw liabilities and contribution levels, the number of employees does not affect the results.
* Employees begin work when they are 30 years old and retire at 65.
* The annual retirement benefit is calculated as (years of service) × (average salary of the last five years) × (a benefit factor of two percent).
* Salaries growth is six percent (actual growth).
* Employees immediately vest.
* Retirement benefits are paid as life annuities (i.e., retirees receive a fixed monthly retirement benefit payment that continues until their death).
* Only the core pension benefit is provided. Other benefits are not included, such as death benefits, survivor benefits, disability benefits, and early retirement benefits.
* The cost method is Entry Age Normal.
* There is a uniform age distribution among the current employees (between 30 and 65) and a uniform age distribution among the retirees.
* Asset holdings are sufficient to cover 85.4 percent of the liability (the median observed funding ratio).

We follow Winklevoss’s (1993) method to calculate normal costs, UAAL costs, and AALs (see appendix) and perform our calculations in Microsoft Excel with Visual Basic coding. We use those calculations to identify the funding ratio (assets/liabilities × 100%) and contribution rates ((normal costs + UAAL costs)/payroll) for the median plan. (The rest of the paper uses the term ARC to refer to the contribution rate calculation.) We then calculate the funding ratio and ARC as actuarial inputs change away from the median condition. We do not focus on the interaction of changing multiple actuarial inputs at the same time.

Table 3 compares the results of the median plan with the conditions that are observed in the actual plans within the PPD.

<Table 3>

The simulation results closely match actual observations with some minor deviations. The simulated normal cost is slightly under-estimated. This is expected since we only consider core pension benefits (e.g., no retirement incentives or disability benefits). The simulated ARC and AAL both fall between the mean and median values of the observed data. The results of this comparison provide reasonable confidence that our simulation is externally valid.

We then create two additional plans whose active employees do not have a uniform age distribution and reset all the actuarial inputs to their median conditions. One of the additional plans is a “young” plan. The age distribution of its active employees is skewed to the right (skewness is approximately .7). The other is an “old” plan. The age distribution is skewed to the left (skewness is approximately -.7). We then calculate the effects of marginal changes in actuarial inputs on the funding ratios and ARCs of the "young" and "old" plans.

Then, we simulate the median plan (with a uniform age distribution) under two additional funding conditions. In the first, assets are set to 70 percent of liabilities (i.e., a 70 percent funding ratio). In the second, assets are set to 100 percent of the liability. We then calculate the effects of marginal changes in actuarial inputs on the funding ratios and ARCs at both of those funding levels.

Results

In this section, we discuss the marginal effects of changes to discount rates, salary growth rate assumptions, mortality tables, cost methods, and amortization periods. Figures 1 and 2 provide a summary view of the range of funding ratios we found by making marginal changes to the actuarial inputs of the median plan while holding all other inputs constant at their median values. In both figures, the circle in the vertical bars represents the median funding ratio and median ARC, respectively. The results illustrate the significant range of effects on the funding ratio and ARC as a result of changing the discount rate and the significant range of effects that is produced on the ARC by changing the salary growth rate assumption. The other effects are also meaningful though they do not create the same range of potential changes.

<Figure 1>

<Figure 2>

The results of marginal changes on funding ratios are shown in table 4 and the results on ARCs are presented in table 5. Those tables show the marginal effects on the median plan in column 1. They present the effects on a plan with a younger demographic in column 2 and an older demographic in column 3. The only difference between columns 1, 2, and 3 are the age demographic of their participants; assets in each of those columns are fixed to the same level, so that the funding level at the median condition is 85.4 percent. Column 4 provides information on the median plan when its asset level is fixed to 70 percent of its liability, and column 5 fixes the assets to 100 percent of its liability. The rest of this section provides analysis on each of the actuarial inputs.

<Table 4>

<Table 5>

**Marginal Changes to the Discount Rate**

We changed the discount rate by .25 percentage point increments to measure its effect on funding ratios (table 4, panel A) and ARCs (table 5, panel A). As expected, we find that funding ratios and ARCs are both highly sensitive to changes in the discount rate. In the median plan (column 1), a one-percentage point increase in the discount rate increases the funding ratio by an average of 9.11 percentage points, and reduces the ARC by an average of 8.56 percentage points.

The median plan’s funding ratio goes from 85.4 percent to 67 percent when the discount rate shifts from its median value (8 percent) to 6 percent, and then up to 92.7 percent funded with a discount rate of 8.75 percent. That is a swing of approximately 25 percentage points without changing any inputs other than the discount rate.

Under the median condition, the ARC is 18.55 percent (of payroll), but it goes up to 36 percent when the discount rate is 6 percent and drops to 12.5 percent with a discount rate of 8.75 percent. About two-thirds of the effect on the ARC comes from changing the UAAL cost. If the median plan has a covered payroll of $2.4 billion (the median annual payroll in the PPD and size that is similar to the annual covered payroll of the Georgia Employee Retirement System, the New Mexico Teachers System, and the New Hampshire Retirement System), its annual contribution at a 6 percent discount rate is about $864 million, at an 8 percent discount rate it is about $445 million, and at 8.75 percent it is about $300 million. This shows the significant incentive for plan sponsors to select and keep a high discount rate. It also shows why plan sponsors often push back on efforts to adopt so-called riskless rates of return to preserve budget resources.

The influence of the discount rate is also affected by a plan's age structure and its asset levels, as shown in columns 2-5.

The discount rate has a larger effect on a younger plan than an older one. This is because active employees in younger plans are further away from their retirement age. The difference between the younger and older plans is smaller when discount rates are higher. At a 6 percent discount rate, the median plan with a young age distribution has an ARC that is about 2.26 percentage points higher, which increases the contribution by $54 million for a plan with a median-sized payroll ($898 million compared to $844 million). As such, there is a larger incentive for plans with younger employees to select and retain higher discount rates.

Also, the effect of changes in the discount rate is larger on well-funded plan than on poorly-funded plans. If the median plan is 100 percent funded with an 8 percent discount rate and then changes its discount rate to 6 percent, its funding ratio will decrease by 20.7 percentage points to 79.7 percent. In contrast, if the median plan is 70 percent funded with an 8 percent discount rate and then changes its discount rate to 6 percent, its funding ratio will decrease by just 14.49 percentage points to 55.51 percent. This is because changing the discount rate affects the denominator in the funding ratio calculation, and increasing the denominator has a bigger effect when the numerator is larger.[[1]](#footnote-1) This relationship is also found in the effect on the ARC. This suggests that well-funded plans actually have more to lose by reducing their discount rates. It also suggests that there is a greater incentive to increase the discount rate by plans that are relatively better funded.

**Marginal Changes in Salary Growth Rate Assumptions**

Panel B of tables 4 and 5 illustrates the effect of changing the salary growth rate assumption. The salary growth rate has a larger influence on the ARC than the funding ratio. Under median conditions, a one percentage point increase in the salary growth rate reduces the average funding ratio by 1.62 percentage points and increases the ARC by 5.35 percentage points.

There are several interesting dynamics that affect the policy implications of salary growth rate assumptions.

The first interesting result is that increasing the salary growth rate assumption has a negative and non-linear effect on the funding ratio. As the salary growth rate assumption increases to the highest observed levels, funding ratios actually increase for all the conditions in table 4, except for the plan with an older age distribution, and even the older plan is close to reversing the downward trend at the 8.00 percent salary growth rate assumption. This quadratic trend is explained by the way EAN allocates the normal cost of employees over their entire career and the relatively greater effect that discount rates have on employees who are many years away from retirement.

Figure 3 illustrates the interactive effect of changes to salary growth rates under different cost methods and at a lower discount rate. The line graphs in the top row illustrate the effect of changing the salary growth rate assumption on funding ratios. The top left graphic reflects the conditions in Panel B of table 4. The top middle line graph shows the effects of the salary growth rate assumption under the PUC cost method. The top right graphic illustrates the effects at a lower discount rate (6 percent). The bottom row of figure 3 illustrates the effect of changing the salary growth rate assumption on the ARCs. Increasing the salary growth rate assumption, regardless of cost method or discount rate, leads to higher contribution requirements, though the magnitude of those changes is affected by the cost method and discount rate.

<Figure 3>

Second, plans that are 100 percent funded, unsurprisingly, pay a lower ARC than those that are 85.74 percent funded and even lower still than those that are 70 percent funded. As salary growth assumptions change, the difference between funding levels remains relatively constant and the differences between ARCs is entirely constant. Therefore, there is little difference in the effect of changing salary growth rates on plans with different asset levels.

Third, demographic conditions, however, affect the influence of changing salary growth rate assumptions. The funding ratio for the “young” plan does not change very much within the full range of observed salary growth rate assumptions under the EAN cost method. Figure 3 demonstrates that this is because of the way the EAN method allocates costs. The line graph in the top-center of figure 3 and the line graph in the top-left are only different in the selection of the cost method. The PUC method presents a clear relationship between the funding ratio and salary growth rate assumption; however, the EAN cost method creates a quadratic relationship that is affected by the demographic conditions of the plan. Younger plans appear to have a much better financial condition with higher salary growth rate assumptions.

Another interactive effect of demographic conditions is that the ARC for the older plan increases at a steeper rate than the ARC for the young plan. This is because there is less time to discount the effect of the assumed salary increase and because of the larger unfunded liability that is created by allocating the new normal cost over the entire career of employees who are near retirement. This means that plans with a large number of participants that are nearing retirement have a larger incentive to lower their salary growth rate assumptions. That change may be particularly attractive since (1) there is significantly less media attention to salary growth rate assumptions, (2) the change is not likely to be noticed in funding ratios (which are often more closely monitored), and (3) the effect on the budgetary costs of pensions is significant. To illustrate, if a plan with an older demographic reduces its salary growth rate assumption from 6.5 percent to 5 percent, it decreases its ARC from 23.92 percent to 14.68 percent. With a median payroll ($2.4 billion), that amounts to a budgetary savings of $221.8 million.

**Actuarial Cost Methods**

PUC and EAN are the most commonly used actuarial cost methods. The decision to choose one or the other has a relatively small marginal effect on funding ratios and liabilities under median conditions, especially in comparison to the much larger effects from discount rates and salary growth rates. As shown in Panel C of tables 4 and 5, the median plan’s funding ratio increases by 4.28 percentage points when it changes to the PUC method and its ARC increases by 0.31 percentage points.

The new GASB standards require that plans report their funding ratios using the EAN method. For most plans currently using PUC, this will likely have a small negative effect on their funding ratios (reducing it by about 3.5 to 5 percent). The effect on the contribution rate is more interesting. Plans with a younger demographic will likely experience a slight increase in their ARCs (about 1.46 percent), and plans with an older demographic will have a lower ARC (about 1.37 percent). This is because EAN accumulates an employee’s normal costs faster in the early years of service, while PUC accumulates normal costs faster in later years. PUC accelerates the accumulation of AAL in the last 20 years before retirement. Both methods reach the same AAL at retirement. These are not large changes and will likely not greatly affect the decision to change the way plans decide how to calculate their contribution rate. However, as demonstrated in figure 3, a more important effect of changing actuarial cost methods is the result of the interactive effect of cost methods with salary growth rate assumptions.

**The Mortality Table**

As shown in Panel D of tables 4 and 5, updating the median plan’s mortality table from RP-2000 to RP-2000 (with a 10-year projection using scale AA) reduced its funding ratio by 2.28 percentage points and increased its ARC by 1.74 percentage points. Further updating the mortality table to RP-2014 reduces the funding ratio by another 4.78 percentage points and increases the ARC by another 3.97 percentage points. This demonstrates why governing boards of some pension systems may delay updating their mortality tables. Given the relatively larger increase to the most recent mortality table, we may expect some delay in adopting the newest table. The decision to update a plan under median conditions from RP-2000 to RP-2014 means reducing the funding ratio by between around 6 to 8 percentage points. The effect is worse for plans that are better funded and younger plans. Mortality tables do not significantly affect the median plan’s ARC, though the younger plan did experience a greater increase in its ARC as a result of updating its mortality table compared to the plans with uniform and older age distributions.

The mortality table affects pension costs and liabilities in three ways. First, before retirement, the mortality table is used to predict the probability that workers will “survive” to the normal retirement age. If employees fail to accumulate sufficient service years to qualify for retirement benefits, their benefits will be eliminated or reduced. Second, updating the mortality rate increases the life expectancy. With longer life expectancies, benefit annuities are paid over longer time periods, so projected pension liabilities increase. The third reason is specific to the changes in individual mortality tables. When you compare RP-2014 with RP-2000, the mortality improvement rate (i.e., the difference in mortality rates from one mortality table to another within the same age group) across all the age groups is approximately 2 percent. However, the improvement is larger for age groups around 40 (a mortality improvement rate of 3 percent) and 65 (a mortality improvement rate of 3.3 percent). If a plan has more employees or retirees at these ages, the plan receives larger effects from updating their mortality table.

**Lengthening the Amortization Period**

As shown in figure 4, when the median plan increases its amortization period from 10 to 20 years, the ARC is reduced by 4.39 percent. That accounts for a $105 million reduction in budgetary costs for a plan with a median-sized payroll. Once the amortization period reaches 40 years, a ten year increase has a very small effect (less than one percent of payroll). This is because longer amortization periods spread the unfunded liability payment over many years.

<Figure 4>

There is no unfunded liability when a funding ratio is 100 percent, so there is no effect from re-amortizing the liability. Sponsors only benefit from re-amortizing when they have an unfunded liability, and the size of the benefit increases as unfunded liabilities increase. If assets cover 90 percent of the liability, the UAAL is small and the length of amortization period has a relatively smaller effect on the ARC. Increasing the amortization period from 10 to 20 years for a plan that is 90 percent funded reduces the ARC by 3 percentage points. When a plan is 60 percent funded, the selection of the amortization period is more important. At 60 percent funded, increasing the amortization period from 10 years to 20 years reduces the ARC by 12 percentage points (equivalent to $288 million in budget savings for a plan with a median-sized payroll).

Discussion and Conclusion

The financial condition of public pensions is an important policy concern in many state and local governments. Those concerns, and the policy debates to which they contribute, are significantly affected by how we measure the liabilities and contribution requirements of public pensions. This paper uses a simulation research design to examine how the selection of different actuarial inputs affects the reported size of pension liabilities (presented as funding ratios) and contribution requirements (presented as annual required contribution as a share of payroll). The results of the simulation illustrate that discount rates, salary growth rate assumptions, cost methods, mortality tables, and amortization periods all greatly influence funding ratios and contribution requirements.

The findings suggest that all forms of actuarial inputs deserve further attention from policy researchers and should be controlled for in empirical research. As expected, funding ratios are especially affected by changes in the discount rate. Contribution requirements are influenced by changes in discount rates but are even more affected by changes in salary growth rate assumptions. Most plans, however, are net expected to change their discount rates or salary growth rates by the full range of values. As such, we looked at smaller marginal changes to discount rates or salary growth rate assumptions (such as .25 or .50 percentage point changes), the effect of those changes on funding ratios and contribution requirements is similar, if not smaller, than the effect of changing cost methods or updating mortality tables.

The influence of actuarial inputs, other than the discount rate, also shows that there are many ways to influence the reported condition of public pension systems without actually changing the fundamental financial health of the plan. Many of those actuarial input changes do not receive significant outside review. Governments and citizen groups should consider conducting independent reviews of the actuarial inputs used to value their pension liabilities to ensure that trustees are selecting actuarial inputs that do not put future taxpayers at risk.   
 This research also demonstrates that plan characteristics have significant interactive effects with actuarial inputs. The demographic profile of pension systems is particularly important. When employees are further from retirement, their true pension benefit is less certain. This simulation show that changes to actuarial inputs for younger employees often has a greater effect than for older employees, except for the expected salary growth rate.[[2]](#footnote-2) This may suggest that some of the decline in funding ratios over the past decade may be related to more public workers approaching retirement and, therefore, a more accurate representation of pension liabilities.

Also, the simulation shows that plans with more assets are likely to experience larger changes in their funding ratios and ARCs as the result of changes to their actuarial assumption. This finding goes against common assumptions that selecting more conservative actuarial inputs is financially easier for well-funded plans to select more conservative actuarial inputs. This suggests the need to examine differences in the management of well-funded and poorly-funded pension systems.

Another contribution of this research is to move us closer to understanding how to compare pension systems in cross-sectional research. Previous studies compare pension costs and funding ratios across pension plans without adjusting or considering the different actuarial assumptions. This study shows that such comparison is very problematic without controlling for actuarial inputs. And, even with controlling for actuarial inputs, this study shows that a general approach to standardize pension funding ratios based on reported data is currently out of reach until we better understand and have better data on plan characteristics, such as age distributions and asset levels. A consistent way of financial reporting, instead, could facilitate comparison of financial conditions across pension plans.

There are several limitations in this study that warrant further research. For one, this study uses a variety of simplifications. We only, for example, look at five actuarial inputs, three asset levels, three demographic conditions, a single benefit plan, and limit beneficiaries to employees. And, we only looked at a few interactive effects. Future research should include more interactive effects, more conditions, additional benefits, and more actuarial inputs.

Another limitation is that this research only considers the marginal effect at the moment that the change is made (all other conditions held constant). More research is needed into the long-term effect of changing actuarial inputs. And, given that the simulation results demonstrate the importance of age demographics, more research is needed in how overoptimistic actuarial inputs affect pension systems as their workers near retirement. Research into long-term effects can also help us better understand the intergeneration equity concerns created by errors (both positive and negative) in actuarial assumptions.

A third limitation is that simulation is based on median conditions. This means that the simulated plan may reflect a typical plan but not provide sufficient insight into specific pension systems. Further work to model actual pension systems will help researchers understand the political incentives and constraints facing actual governments and actual pension managers.

Finally, this paper has treated funding ratios and contribution effects as equivalent measures. Much of the popular discussion on the financial health of pension systems focuses on the funding ratio (or a similar stock measure). However, while funding ratios and contribution requirements are associated, they are not the same. Whether pension systems are 72 or 79 percent funded, for example, is unlikely to influence the political environment or the opinions of most credit raters. However, a small increase in the contribution rate can create significant budgetary pressure or further strain governments experiencing fiscal strain. Further research is needed into how public officials and stakeholders view the relative importance of funding ratios and contribution requirements.

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Appendix

Normal costs (NC), accrued actuarial liabilities (AAL), funding ratios, and annual required contributions (ARC), under different actuarial cost methods, are calculated by the following formulas:

is the normal cost at the age of *x* with the assumed retirement age of *r* under EAN method.

is the normal cost at the age of *x* with the assumed retirement age of *r* under PUC method.

is the AAL at the age of *x* with the assumed retirement age of *r* under EAN method.

is the AAL at the age of *x* with the assumed retirement age of *r* under PUC method.

*x* is the current age, which varies according to different employees at a certain year.

*y* is the entry age, which is assumed to be 30.

*r* is the retirement age, which is assumed to be 65.

is the present value of all future benefits at the age of x with the assumed retirement age of *r*. The value is calculated based on the discount rate (discount factor) and the expected salary growth rate.

is the salary at the age of *x*.

is the salary at the entry age *y*.

is the probability of surviving (*x*-*y*) years from the entry age *y* to the current age *x*.

is the discount factor from entry age *y* to the current age *x*.

is a salary-based annuity factor, which represents the present value of an employee’s future salary from the entry age *y* to retirement age *r*, per unit of salary at age *y*.

is a salary-based annuity factor, which represents the present value of an employee’s future salary from the entry age *y* to current age *x*, per unit of salary at age *y*.

is the funding ratio for a plan at *t* time period.

is the asset value for a plan at *t* time period. The value of the asset is initially fixed as a defined portion of the liability. We keep the asset value fixed at *t* time periods.

is the annual required contribution at *t* time period for a plan.

is the normal cost at *t* time period for a plan. Normal cost can be calculated under EAN or PUC.

is the sum of accrual liability for both active members and retirees in a plan.

is an m-year period certain annuity. *m* is the amortization period.

Table 1: Variations in actuarial assumptions

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 1st  percentile | 25th percentile | 50th percentile | 75th percentile | 99th percentile |
| Discount rate | 6% | 7.75% | 8% | 8.25% | 8.75% |
| Salary growth rate | 3.1% | 5% | 5.68% | 6.25% | 8% |
| Amortization period | 1.32 | 20 | 29 | 30 | 100 |

Table 2: Variations in using actuarial cost methods

|  |  |
| --- | --- |
|  | Incidence in Public Pension Plans |
| Entry Age Normal (EAN) | 72% |
| Projected Unit Credit (PUC) | 13% |
| Aggregate Cost (AGG) | 8% |
| Other methods (e.g., frozen initial liability) | 7% |

Table 3: Comparing the simulation results to observed conditions in actual public pensions

|  |  |  |  |
| --- | --- | --- | --- |
|  | Test results when assumptions are set to the median conditions the PPD (see note) | Observed Conditions in the PPD data | |
| Median | Mean |
| Normal Cost | 10.2 | 12.0 | 13.0 |
| Actuarially Determined Contribution (ADC) | 18.5 | 18.1 | 22.2 |
| Actuarially Accrued Liability (AAL) | 6.4 | 5.0 | 6.7 |
| Note: All numbers reported as percent of payroll. The simulated results are based on the typical actuarial inputs (median values) from the PPD: benefit multiplier = 2%, discount rate = 8%, salary growth rate = 5.68%, amortization period = 29 years, the mortality table is the RP-2000 with 10 year projection, and the cost method is EAN. The simulation also assumes a uniform age distribution in the active workers and retirees. | | | |

Table 4: Effect of actuarial input changes on funding ratios

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Actuarial Input** | **Assets in median condition are set to**  **an 85.4% funding ratio** | | | **Assets in median condition set to a 70%**  **funding ratio**  **(4)** | **Assets in median condition set to a 100%**  **funding ratio**  **(5)** |
| **Active participant ages are uniformly distributed**  **(1)** | **Active participant ages are skewed younger**  **(2)** | **Active participant ages are skewed older**  **(3)** |
| **Panel A: Discount Rates (median observed value is 8%)** | | | | | |
| 8.75 | 92.77 | 93.35 | 92.42 | 76.04 | 108.63 |
| 8.50 | 90.27 | 90.64 | 90.04 | 73.99 | 105.70 |
| 8.25 | 87.81 | 87.99 | 87.70 | 71.98 | 102.82 |
| 8.00 | 85.40 | 85.40 | 85.40 | 70.00 | 100.00 |
| 7.75 | 83.04 | 82.86 | 83.14 | 68.06 | 97.23 |
| 7.50 | 80.72 | 80.38 | 80.92 | 66.16 | 94.51 |
| 7.25 | 78.44 | 77.94 | 78.74 | 64.30 | 91.85 |
| 7.00 | 76.21 | 75.57 | 76.60 | 62.47 | 89.24 |
| 6.75 | 74.02 | 73.24 | 74.50 | 60.68 | 86.68 |
| 6.50 | 71.88 | 70.97 | 72.44 | 58.92 | 84.17 |
| 6.25 | 69.78 | 68.75 | 70.42 | 57.20 | 81.71 |
| 6.00 | 67.73 | 66.58 | 68.43 | 55.51 | 79.30 |
| Average effect of 1% increase | 9.11 | 9.73 | 8.72 | 7.46 | 10.66 |
| **Panel B: Salary Growth Rate Assumption (median observed value is 5.68%)** | | | | | |
| 8.00 | 83.75 | 88.61 | 81.07 | 68.64 | 98.06 |
| 7.50 | 83.52 | 86.86 | 81.63 | 68.46 | 97.80 |
| 7.00 | 83.68 | 85.83 | 82.43 | 68.59 | 97.98 |
| 6.50 | 84.14 | 85.34 | 83.44 | 68.97 | 98.53 |
| 6.00 | 84.85 | 85.27 | 84.59 | 69.55 | 99.35 |
| 5.50 | 85.74 | 85.52 | 85.87 | 70.28 | 100.40 |
| 5.00 | 86.78 | 86.03 | 87.24 | 71.13 | 101.62 |
| 4.50 | 87.94 | 86.74 | 88.68 | 72.08 | 102.98 |
| 4.00 | 89.19 | 87.60 | 90.17 | 73.11 | 104.44 |
| 3.50 | 90.50 | 88.59 | 91.70 | 74.18 | 105.98 |
| 3.00 | 91.86 | 89.67 | 93.24 | 75.30 | 107.57 |
| Average effect of 1% increase | -1.62 | -0.21 | -2.43 | -1.33 | -1.90 |
| **Panel C: Cost Method (EAN is the most commonly observed type)** | | | | | |
| EAN | 85.40 | 85.40 | 85.40 | 70.00 | 100.00 |
| PUC | 89.68 | 90.03 | 89.48 | 73.51 | 105.02 |
| **Panel D: Mortality Tables (RP-2000 with projection is the most commonly observed type)** | | | | | |
| RP-2000 | 87.68 | 87.88 | 87.57 | 71.87 | 102.68 |
| RP-2000 with projection | 85.40 | 85.40 | 85.40 | 70.00 | 100.00 |
| RP-2014 | 80.62 | 80.26 | 80.84 | 66.08 | 94.41 |

Note: Shaded cells all represent the uniform plan at median conditions (near median for salary growth rate). All values are funding ratios (percent of liabilities that are covered by asset holdings).

Table 5: Effect of actuarial input changes on the ARC/Payroll

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Actuarial Input** | **Assets in median condition are set to**  **an 85.4% funding ratio** | | | **Assets in median condition set to a 70%**  **funding ratio**  **(4)** | **Assets in median condition set to a 100%**  **funding ratio**  **(5)** |
| **Active participant ages are uniformly distributed**  **(1)** | **Active participant ages are skewed younger**  **(2)** | **Active participant ages are skewed older**  **(3)** |
| **Panel A: Discount Rates (median observed value is 8%)** | | | | | |
| 8.75 | 12.50 | 12.30 | 12.61 | 21.91 | 3.57 |
| 8.50 | 14.49 | 14.43 | 14.53 | 23.70 | 5.76 |
| 8.25 | 16.51 | 16.58 | 16.47 | 25.51 | 7.98 |
| 8.00 | 18.55 | 18.77 | 18.43 | 27.35 | 10.21 |
| 7.75 | 20.62 | 20.98 | 20.41 | 29.21 | 12.48 |
| 7.50 | 22.72 | 23.22 | 22.42 | 31.11 | 14.77 |
| 7.25 | 24.85 | 25.49 | 24.46 | 33.04 | 17.08 |
| 7.00 | 27.01 | 27.80 | 26.54 | 35.00 | 19.43 |
| 6.75 | 29.20 | 30.15 | 28.64 | 37.00 | 21.81 |
| 6.50 | 31.44 | 32.54 | 30.78 | 39.04 | 24.23 |
| 6.25 | 33.71 | 34.96 | 32.96 | 41.12 | 26.68 |
| 6.00 | 36.02 | 37.44 | 35.18 | 43.25 | 29.18 |
| Average effect of 1% increase | -8.56 | -9.14 | -8.21 | -7.76 | -9.73 |
| **Panel B: Salary Growth Rate Assumption (median observed value is 5.68%)** | | | | | |
| 8.00 | 34.58 | 29.23 | 37.77 | 43.38 | 26.24 |
| 7.50 | 30.36 | 26.77 | 32.50 | 39.16 | 22.02 |
| 7.00 | 26.61 | 24.43 | 27.91 | 35.41 | 18.27 |
| 6.50 | 23.27 | 22.19 | 23.92 | 32.07 | 14.93 |
| 6.00 | 20.29 | 20.06 | 20.43 | 29.09 | 11.95 |
| 5.50 | 17.63 | 18.06 | 17.37 | 26.42 | 9.29 |
| 5.00 | 15.24 | 16.17 | 14.68 | 24.03 | 6.90 |
| 4.50 | 13.09 | 14.39 | 12.32 | 21.89 | 4.75 |
| 4.00 | 11.16 | 12.73 | 10.22 | 19.95 | 2.82 |
| 3.50 | 9.41 | 11.17 | 8.36 | 18.21 | 1.07 |
| 3.00 | 7.83 | 9.72 | 6.71 | 16.63 | -0.51 |
| Average effect of 1% increase | 5.35 | 3.90 | 6.21 | 5.35 | 5.35 |
| **Panel C: Cost Method (EAN is the most commonly observed type)** | | | | | |
| EAN | 18.55 | 18.77 | 18.43 | 27.35 | 10.21 |
| PUC | 18.87 | 17.31 | 19.80 | 27.66 | 10.53 |
| **Panel D: Mortality Tables (RP-2000 with projection is the most commonly observed type)** | | | | | |
| RP-2000 | 16.81 | 16.88 | 16.76 | 25.60 | 8.47 |
| RP-2000 with projection | 18.55 | 18.77 | 18.43 | 27.35 | 10.21 |
| RP-2014 | 22.52 | 23.04 | 22.22 | 31.32 | 14.19 |

Notes: Shaded cells all represent the uniform plan at median conditions (near median for salary growth rate). All values reported as percent of covered payroll.

Figure 1: Funding ratio values at the full range of changes to each actuarial input (ceteris paribus)

Figure 2: ADC/payroll values at the full range of changes to each actuarial input (ceteris paribus)

Figure 3: Comparison of the effects of changing salary growth rate assumptions under different conditions

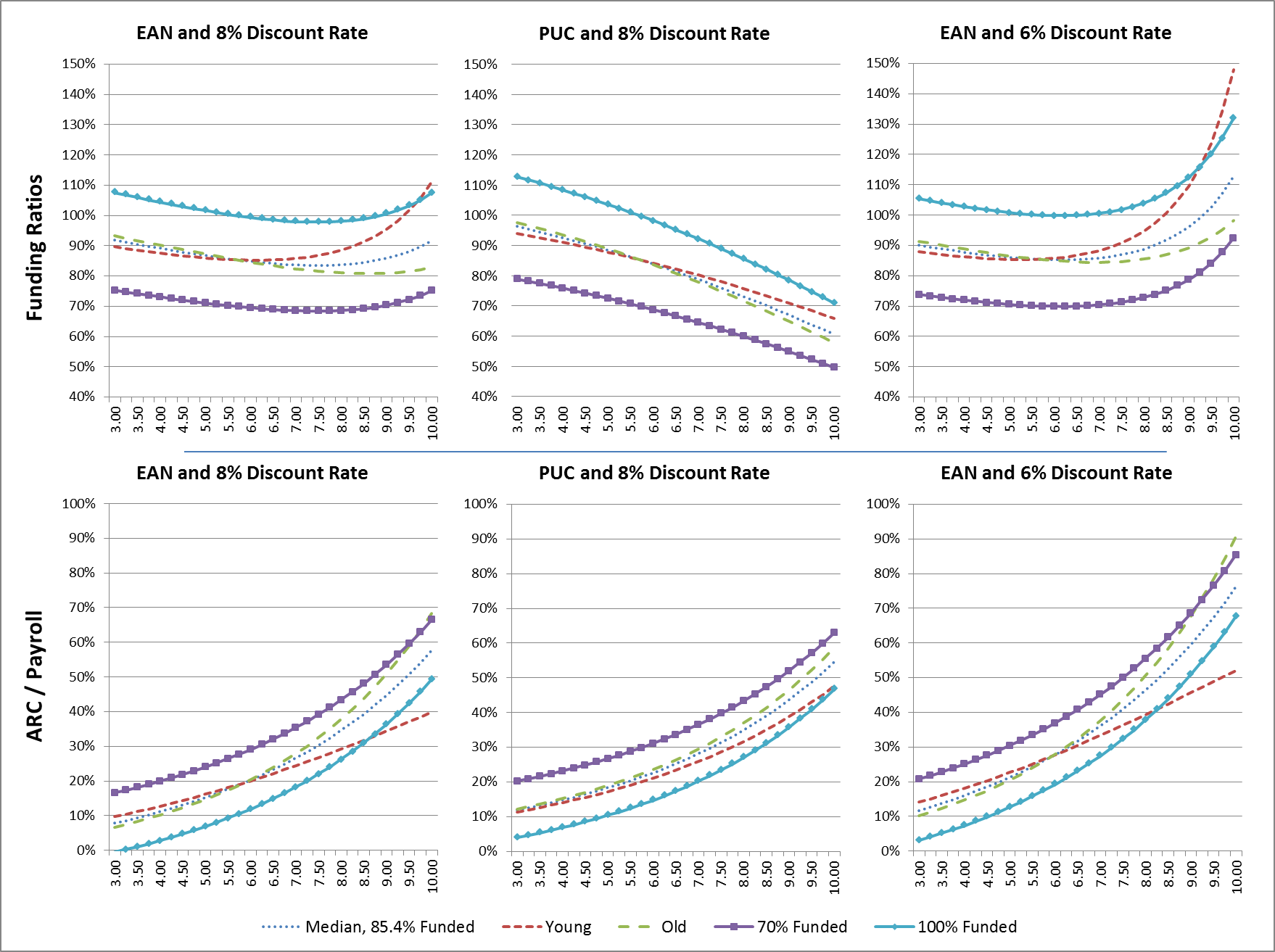
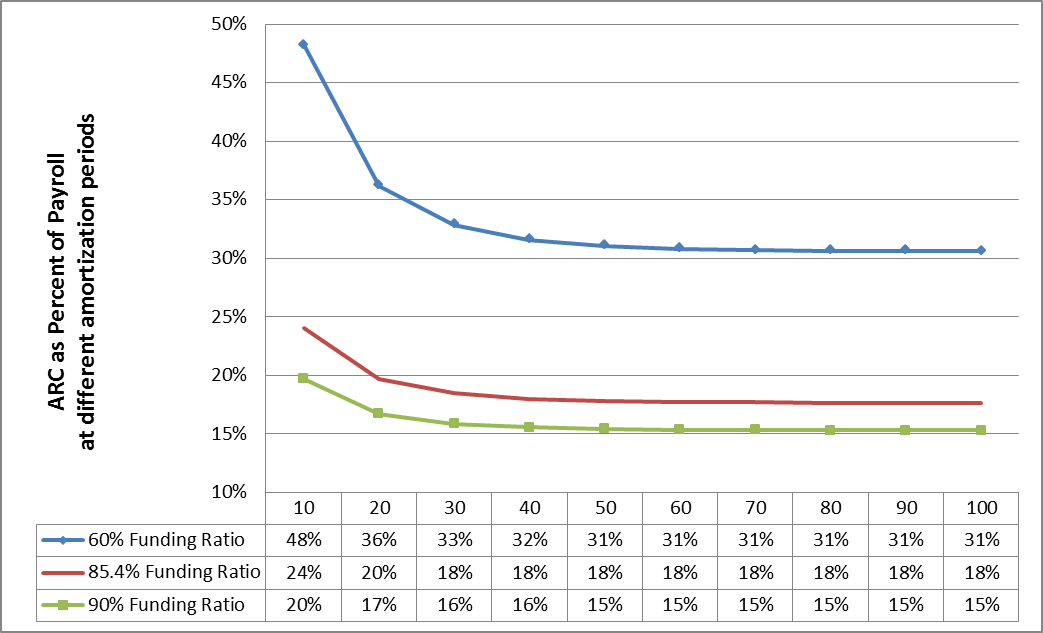


Figure 4: Effect of changing the amortization period on the ARC/Payroll



1. For example, if a plan is 100 percent funded with $100 assets and $100 liabilities, a 10 percentage point decrease in the liability increases the funding ratio by 11.11 percentage points ($100/$100-$100/$90)\*100%. And, if a plan is 50% funded with $50 assets and $100 liabilities, a 10% decrease in the liability only increases the funding ratio by 5.56 percentage points ($50/$100-$50/$90)\*100%. [↑](#footnote-ref-1)
2. As explained in the last section, because EAN accumulates AALs at different pace for different ages, when a plan changes the expected salary growth rate, an older plan is effected more than a younger plan. [↑](#footnote-ref-2)